METAHEURISTICS FOR SOME GRAPH THEORETIC OPTIMIZATION PROBLEMS IN CRISP AND FUZZY ENVIRONMENT

A Synopsis

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Stuti Srivastava

Dr. Richa Bansal	Dr. Antika Thapar	Prof. Ravinder Kumar
Supervisor	Co-Supervisor	Head, Department of Mathematics
Department of Mathematics	Department of Mathematics	Dean, Faculty of Science

Department of Mathematics

Faculty of Science

Dayalbagh Educational Institute, Agra

(Deemed University)

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1. Introduction -

Graph Theory, a branch of Mathematics, is a study and analysis of graphical structures to solve some optimization problems. When a real life problem becomes very complicated to formulate or cannot be solved by classical methods then it can be modeled as a graph which are helpful in finding an approximate solution of the problem. Graph theory has considerable applications in Operations Research, Computer Sciences, Chemistry, Microbiology, Linear Algebra, Economics, Medicine, Topology etc. Many problems related to these subjects can be solved by graph theory. Some examples of these include cloud computing, map colorings, web graphs, physical networks, signal-flow graphs, tracing mazes as well as less tangible interactions occurring in social networks and in a flow of a computer program.

The idea of graph theory came in the year 1735 for the first time when a paper published in Clarendon Press by a Swiss mathematician Leonhard Euler, based on the problem of Konigsberg bridge in the city of Konigsberg formerly in Germany. After this problem, graph theory has been used to solve many real world problems which are NP hard. For example, the matrix bandwidth minimization problem (MBMP) which consists of permuting the row and columns of a 0-1 sparse symmetric matrix to keep the non zero elements in a band that is as close as possible to the main diagonal. In mathematical terms it is defined as: Given a 0-1 sparse symmetric matrix $A = \{a_{ij}\}$, the bandwidth of A is $B(A) = \max\{|i-j| : a_{ij} \neq 0\}$. The MBMP consists of finding symmetric permutation A' of A such that $B(A)$ is minimized. This problem is equivalent to the graph bandwidth minimization problem which is stated as follows: Given a finite undirected graph $G = (V, E)$, where V is the set of vertices, E is the set of edges and $f : V \to \{1, 2, ..., n\}$ is the labeling of the vertices then the bandwidth of G for f is defined as $B_f(G) = \max\{|f(i) - f(j)| : (i, j) \in E\}$. The bandwidth minimization problem for graph G is to find a labeling f for which $B_f(G)$ is minimum. Many such problems, which are NP hard that cannot be solved by any exact (deterministic) method, are modeled as graph theoretic optimization problems. Since, in most of the cases the exact solution of such problems is not required, so metaheuristic techniques have been introduced in graph theory. For better understanding of metaheuristics, they are defined precisely in the next section.

2. Metaheuristics - The term meta-heuristics was introduced by Glover [GF] and represents a class of promising algorithms for solving hard optimization problems. The meta-heuristic algorithms are aiming at efficient and comprehensive exploration of the search space, using the governing mechanisms which imitate certain strategies taken from nature, social behavior, physical laws, etc., in order to find near optimal solutions.

In last thirty years metaheuristics became a topic of great interest. This is the reason that a number of metaheuristic techniques have been developed and applied to large number of

problems. Metaheuristics are of two types: single solution based and population based. Some examples for each category are given below:

(a) Single solution based metaheuristics

Simulated Annealing
Tabu Search
Greedy Random Adaptive Search Procedure
Variable Neighborhood Search

(b) Population based metaheuristics

Genetic Algorithm
Memetic Algorithm
Ant Colony Optimization
Particle Swarm
Artificial Bee Colony Optimization

A brief introduction of these methods is as follows:

Single solution based metaheuristics -

2.1 Simulated Annealing - Simulated annealing (SA) method is inspired by physical annealing process. When a metal is heated above its melting point, holding this temperature and then cooling it slowly until it comes again in its solid form, this procedure is called annealing. When a metal is cooled slowly it attains a perfect crystalline structure which possesses minimum energy while when it is cooled fast it attains its maximum energy [DS]. Kirkpatrick et al. in 1983 [KGV] and Cerny in 1985 [C] observed this process and gave a method to solve optimization problems where objective is to minimize a function. SA is applicable to mixed discrete as well as continuous problems and its formulation is also very easy. The benefit of this method is that it is able to escape from local minima and its space and time complexity is less than genetic algorithms [GF]. Because of these advantages this method became very popular since the end of 1980s. The algorithm starts by generating an initial solution and by initializing the temperature parameter T. Then, at each iteration, a solution s_0 is randomly selected in the neighborhood $N(s)$ of the current solution s. The solution s_0 is accepted as new current solution depending on T and on the values of the objective function for s_0 and s, denoted by $f(s_0)$ and $f(s)$, respectively. If $f(s_0) \leq f(s)$, then s_0 is accepted and it replaces s. On the other hand, if $f(s_0) > f(s)$, s_0 can also be accepted with some probability. Outline of an SA algorithm is given below:

1 *Choose an initial solution s randomly for the system to be optimized*
2 *Initialize the temprature T*
3 ***While*** *the stoping criterion is not satisfied* ***do***

```
    repeat
        Randomly select s' ∈ N(s)
        if f(s') ≤ f(s) then
            s ← s'
        else
            s ← s' with a probability p(T, f(s'), f(s))
        end
    until the thermodynamic equilibrium of the system is reached
    Decrease T
end
return the best solution obtained
```

2.2 Tabu Search :- Tabu search (TS) is a metaheuristic that guides a local search heuristic procedure to escape from local minima and to explore the solution space. This method was proposed by Grover in 1986 [GP]. Tabu search is an extension of previously existing local search approaches. In fact it can be seen that basic tabu search is a combination of local search with short term memories [BLS]. The process starts with initializing a random solution. A candidate list of moves is created by applying some specific steps. In each iteration, each move from candidate list generates a new solution from current solution. This set of solutions is regarded as tabu list and a solution from it is recorded as best solution if it improves the previous best and tabu list is updated. The process stops if specific number of iterations has elapsed in total or the last best solution is found. The basic algorithm of tabu search is as follows:

```
1. Choose at random an initial solution s in the search space
2. Tabu List ← ∅
3. while the stopping criteria is not satisfied do
4.     select the best solution s' ∈ N(s)\ Tabu List
5.     s ← s'
6.     update Tabu List
7. end
8. return the best solution met
```

2.3 Greedy Randomized Adaptive Search Procedure - Greedy randomized adaptive search procedure (GRASP) is one of the popular metaheuristics of 1990s which was introduced by Feo and Resende in 1995 [FR]. GRASP is an iterative randomized sampling procedure which consists of two phases: one is construction phase and another is a local search phase [BAS]. The construction phase creates random feasible solutions by applying a greedy procedure and local search phase improves the solution until a local minima is found. The overall best solution is returned as final result. A pseudo code of GRASP is given as follows [FR]:

```
procedure grasp ()
    1 InputInstance () ;
    2 for GRASP stopping criterion not satisfied
```

```
    3 ConstructGreedyRandomizedSolution(Solution) ;
    4 LocalSearch(Solution) ;
    5 UpdateSolution(Solution, Best_Solution_Found) ;
    6 repeat the function;
    7 return(Best_Solution_Found)
end grasp;
```

2.4 Variable Neighborhood Search - Variable neighborhood search (VNS) is proposed by Mladenovic and Hansen in 1997 [BAS]. Contrary to other metaheuristics based on lacal search methods, VNS does not follow a trajectory but explores increasingly distant neighborhood of the current incumbent solution and jumps from this solution to a new one if and only if an improvement has been made [HM]. In this way optimal variable is kept in the incumbent and obtain promising neighbors. VNS procedure can be understood by following algorithm [BLS]:

```
1 Select the set of neighborhood structures N_k, k=1,…., k_max
2 Choose at random an initial solution s in the search space
3 while the stopping criteria is not satisfied do
4 |  n ← 1
5 |   while n < n_max do
6 |     |  shaking : select a random solution s' in the nth neighborhood of s
7 |     |  apply a local search starting from s' to get a random solution s''
8 |     |  if s'' is better than s then
9 |     |     |     s ← s''
10|     |     |     n ← 1
11|     |  else
12|     |     |     n ← n+1
13|     |  end
14|   end
15| end
return the best solution obtained
```

Population based metaheuristics –

Most of the population based metaheuristics follow a similar sequence of operations and therefore can be defined within a common framework as shown in algorithm given below:

1. *Set iteration counter t=0*
2. *Initialize a population S_t of N solutions randomly*
3. *Evaluate each member of S_t*
4. *mark the best solution of S_t as x_t^**
5. ***repeat***
6. *choose n solutions (let P_t) from S_t using a selection procedure*

7. *Generate m new solutions (let C_t) from P_t using a generation procedure*
8. *Choose k solutions (let R_t) from S_t using a replacement procedure*
9. *Update S_t by replacing R_t using k solutions from a pool of any combination of at most three sets P_t, C_t & R_t using an update procedure*
10. *Evaluate each member of S_t*
11. *Identify the best solution of S_t and update x_t^**
12. *$t \leftarrow t+1$*
13. ***Until** a termination criterion is satisfied*
14. *Declare the optimal solution as x_t^**

Some of the population based metaheuristics are described briefly as follows:

2.5 Genetic Algorithm - Genetic algorithms (GA) are the heuristics search and optimization techniques that mimic the process of natural evolution. GA was proposed by John Holland in 1960s [BAS]. It is a type of evolutionary algorithm based on the principle of evolution via natural selection. It acts on a population of individuals that undergo selection in the presence of operators such as mutation and crossover [BAS]. A fitness function is used to evaluate the individuals. Genetic algorithm starts with initializing a random population of chromosomes and a fitness function. At each iteration, a population of individuals is evolved from one generation to the next generation through selection, crossover and mutation. The new population of individuals so obtained, replace the previous one and the overall best solution is updated. This procedure is repeated until the desired number of iterations or the algorithm converges to a solution.

2.6 Memetic Algorithm - Memetic algorithm (MA) was introduced by Moscato in his technical report in 1989 [M]. Memetic algorithms are a class of stochastic global search heuristics in which evolutionary algorithms (EA) based approaches (e.g. GA) are combined with problem-specific solvers such as local search heuristic techniques, approximation algorithms or sometimes even (partial) exact methods. Memetic algorithms have received various names throughout the literature. Some of the alternative names used by scientists are hybrid GAs, Baldwinian EAs, Lamarckian EAs, genetic local search algorithms [MTK]. The method is based on a population of agents and proved very successful in a variety of problem domains and in particular for finding an approximate solution of NP-hard optimization problems. The general structure of MAs is given in Fig. 1 below:

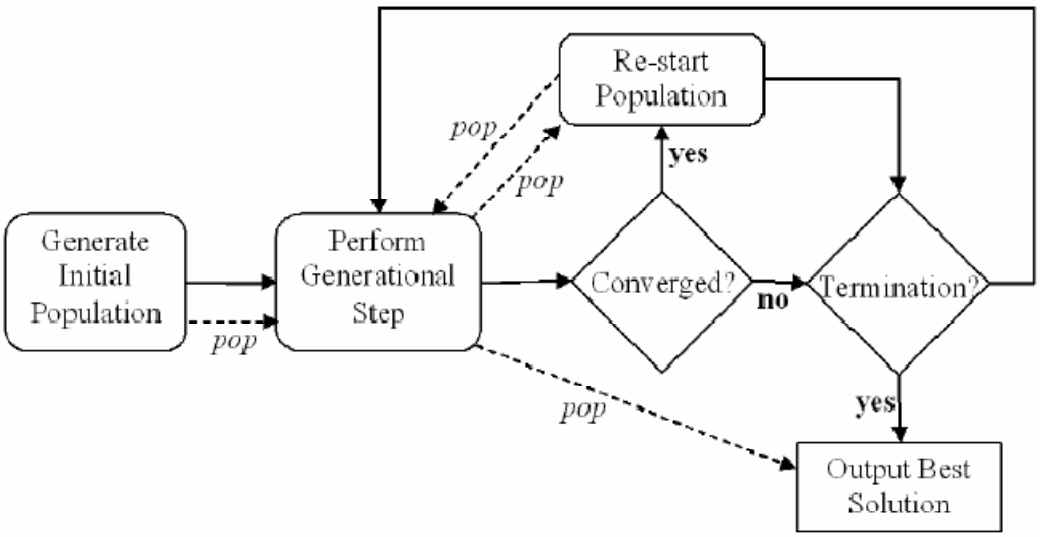

Fig. 1

Here, solid arrows indicate the control flow, whereas dashed arrows indicate the data flow.

2.7 Ant Colony Optimization - Ant colony optimization (ACO) is one of the popular population based metaheuristic optimization technique which was introduced early in 1990s by M. Dorigo and colleagues [B1]. This method was first applied on travelling salesman problem. Later on an improved ant colony optimization technique was also developed and applied on several number of optimization problems. This method is inspired by the foraging (randomized search) behavior of real ants and pheromone communication between them during search of food which enables them to find shortest path between their colony (nest) and food source [DC]. When searching for food, ants explore the surrounding area near their colony by performing randomized walk. During their walk they leave pheromone on the trail in order to mark the trail which would guide other ants to find the food source. Other ants smell the pheromone and follow the same trail. The path by which most of the ants go through becomes the trail of high pheromone. While returning to their nest they follow the path of high pheromone. By this way they find their shortest path from nest to food source. The algorithm starts with a random population and pheromone parameter *P*. In each iteration, the value of pheromone is updated by applying a series of steps. At last a solution with high pheromone value is selected.

2.8 Particle Swarm Optimization - Particle swarm optimization (PSO) is a swarm intelligence metaheuristic proposed by Eberhart and Kennedy in 1995 [RSY]. This method is inspired by the social behavior of animals that don't have any leader in their group or swarm. For example bird flocks that have no leader, find food randomly by following one of the members of the group that has the closest position to a food source (potential solution) [HSE]. Another

example is fish school. Animals find their best condition through communication among members who already have a better solution. Animal which have better condition informs to his swarm and others get the same solution by following him. This procedure repeats until best condition discovered. PSO follow the work of this animal society. Unlike genetic algorithms, PSO does not change its population in each iteration but update the position of members of the population.

2.9 Artificial Bee Colony Optimization - Artificial bee colony (ABC) algorithm was developed by Karaboga [KB]. This method is based on foraging behavior of honey bees. In ABC algorithm, the colony of artificial bees consists of three groups of bees:

- Employed bees
- Onlookers
- Scouts

First half of the colony consists of the employed artificial bees and the second half includes the onlookers. For every food source, there is only one employed bee i.e. the number of employed bees is equal to the number of food sources around the hive [KB]. When bees abandoned the food source of an employed bee, it becomes a scout [SB]. In ABC, artificial bees (employed or onlookers) fly in multi dimensional space and find their food source by their own experience or by following their nest mates and adjust their position. Scout bees find their food source randomly without using their experience. If present amount of food is greater than the previous one, they memorize the new position and forget the previous one. Thus ABC system combines local search methods, carried out by employed and onlooker bees with global search methods, managed by onlookers and scouts, attempting to balance exploration and exploitation process.

3. Pre - Requisites of Graph Theory and Fuzzy Theory -

In this section some basic definitions related to graph theory and fuzzy theory are presented to help understand graph theoretic optimization problems defined in next section.

Graph - A graph $G(V, E)$ is a combination of a set of vertices $V=\{v_1, v_2,\ldots,v_n\}$together with a set of edges $E=\{e_1, e_2,\ldots,e_m\}$, such that each edge e_k is a pair of vertices (v_i, v_j) and is denoted by v_iv_j.

Neighborhood **-** The neighborhood of a vertex v is defined as $N(v)=\{u\in V \mid uv\in E\}$.

Path - A *u-v* path is defined as a sequence of vertices starting at *u* and ending at *v*, where consecutive vertices in the sequence are adjacent vertices in the graph and no vertex is repeated.

Cycle - A cycle is a closed path in which the first and last vertices are the same.

Connected and Disconnected Graph - A graph G is connected if it has a uv-path whenever $u, v \in V$, otherwise G is disconnected.

Tree - A connected graph is said to be tree if it has no cycle.

Edge Weighted Graph - A graph $G(V,E)$ together with a map $\varphi_E: E \rightarrow \mathcal{R}^+$ is an edge weighted graph denoted by $G(V, E, \varphi_E)$.

Bipartite Graph **-** A bipartite graph is one whose vertex set can be divided into two independent sets, V_1 and V_2 such that every edge of the graph connects a vertex in V_1 to one in V_2.

Fuzzy Set - A classical set is a collection of well defined objects with membership grade either 0 or 1. A Fuzzy set is a collection of well defined objects with membership grade between 0 to 1 i.e. [0,1].

Membership Grade - A fuzzy set A in universal set X is characterized by a membership function $\mu_A(x)$ which associates each point in X, a real number in the interval [0,1], with the values of $\mu_A(x)$ at x representing the "grade of membership" of x in A. Thus, the nearer the value of $\mu_A(x)$ to unity, the higher the grade of membership of x in A.

Intuitionistic Fuzzy Set - An intuitionistic fuzzy set (IFS) A in X is given by
$A = \{x, \mu_A(x), \gamma_A(x) \in X\}$ where $\mu_A(x): X \rightarrow [0,1]$ and $\gamma_A(x): X \rightarrow [0,1]$ and for every $x \in X$, $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$.

Intuitionistic Fuzzy Normal Set (IF-normal) - An IFS $A = \{x, \mu_A(x), \gamma_A(x) \in X\}$ is called IF-normal set if there exist at least two points $x_0, x_1 \in X$ such that $\mu_A(x_0) = 1$ and $\gamma_A(x_1) = 1$.

Intuitionistic Fuzzy Convex Set (IF-convex) - An IFS $A = \{x, \mu_A(x), \gamma_A(x) \in X\}$ of the real line is called IF- convex set if $\forall\, x_1, x_2 \in \mathbb{R}$ and $\forall \lambda \in [0, 1]$

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \mu_A(x_1) \wedge \mu_A(x_2)$$
$$\gamma_A(\lambda x_1 + (1-\lambda)x_2) \geq \gamma_A(x_1) \wedge \gamma_A(x_2)$$

Intuitionistic Fuzzy Number - An IFS $A = \{x, \mu_A(x), \gamma_A(x) \in X\}$ of the real line is called an intuitionistic fuzzy number (IFN) if

a) A is IF- normal
b) A is IF- convex
c) A is bounded
d) μ_A is upper semi continuous and γ_A is lower semi continuous

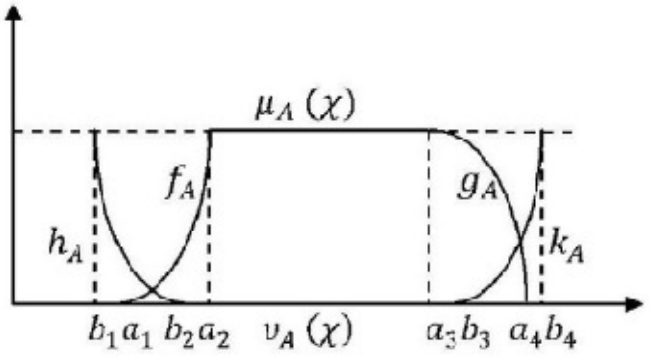

Fig.2: An Intuitionistic Fuzzy Number

From the definition given above it may get at once that for any IFN A there exist eight numbers $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4 \in \mathbb{R}$ such that $b_1 \le a_1 \le b_2 \le a_2 \le a_3 \le b_3 \le a_4 \le b_4$ and four functions $f_A, g_A, h_A, k_A : \mathbb{R} \to [0,1]$, called the sides of a fuzzy number, where f_A and k_A are non decreasing and g_A and h_A are non increasing, such that we can describe a membership function μ_A in form

$$\mu_A(x) = \begin{cases} 0 & \text{if} \quad x < a_1, \\ f_A(x) & \text{if} \quad a_1 \le x \le a_2, \\ 1 & \text{if} \quad a_2 \le x \le a_3, \\ g_A(x) & \text{if} \quad a_3 \le x \le a_4, \\ 0 & \text{if} \quad a_4 < x \end{cases}$$

while a non membership function γ_A has a following form

$$\gamma_A(x) = \begin{cases} 0 & \text{if} \quad x < b_1, \\ h_A(x) & \text{if} \quad b_1 \le x \le b_2, \\ 1 & \text{if} \quad b_2 \le x \le b_3, \\ k_A(x) & \text{if} \quad b_3 \le x \le b_4, \\ 0 & \text{if} \quad b_4 < x \end{cases}$$

4. Graph Theoretic Optimization Problems -

In this section some graph theoretic optimization problems in crisp and fuzzy environment are presented as follows:

4.1 Bandwidth Coloring Problem -

The bandwidth coloring problem (BCP) consists of assigning a color to each vertex of a graph, so that the absolute value of the difference between the colors of adjacent vertices is at least the

value of the weight of the associated edge. This problem generalizes the classical vertex coloring problem. The problem is defined formally as follows:

Let $G = (V, E)$ be a graph with a vertex set V and an edge set E with a strictly positive integer weight d_{ij} associated to each edge $(i, j) \in E$. A k-coloring c of G, labels each vertex $i \in V$ with an integer $c(i) \in \{1, 2, ..., k\}$ (called color) in such a way that $|c(i) - c(j)| \geq d_{ij}$ for all $(i, j) \in E$. The bandwidth coloring problem consists of finding a k-coloring with the smallest value of k. In mathematical terms [MGD]:

$$\begin{aligned} &\text{(BCP): Minimize } k \\ &\text{s.t. } |c(i) - c(j)| \geq d_{ij} && \forall\, (i,j) \in E \\ &c(i) \in \{1,2,\dots,k\} && \forall\, i \in V \end{aligned}$$

The classical vertex coloring problem (VCP) is a particular case of the BCP in which d_{ij}=1 for all $(i, j) \in E$.

4.2 Incremental Graph Drawing Problem -

Layout strategies that strive to preserve perspective from earlier drawings are called incremental. Incremental graph drawing problem involves minimizing the number of arc crossings when new vertices and arcs have been added to the graph. In order to preserve the drawing of the original graph, the relative position among the old vertices is kept as it appears in the previous drawing.

Incremental graph drawing constructions are motivated by the need to support the interactive updates performed by the user. It would not be very intuitive or effective for a user to have a drawing tool in which after a slight modification of the current graph, the resulting drawing appears very different from the previous one and also since “real world” graphs tend to be large, an automated procedure to deal with the arc crossing minimization problem in the context of incremental strategies is desirable [ME].

4.3 Generalized Median Graph Problem -

The generalized median graph of a set S of graphs is a graph that minimizes the sum of its distances to all other graphs in S. The objective of the problem is to find median graph of set S of graphs. In generalized median graph problem, median graph may or may not be belong to the set S. Applications of the problem include 2D and 3D image analysis, document processing, biometric identification, image database, video analysis etc. [MR].

4.4 Crossing Minimization Problem of Drawing Bipartite Graph in One Layer One Cluster -

The crossing minimization problem is a classic and an important problem in graph drawing especially for very large scale graphs in which the readability of the graph depends upon the number of crossings. Lesser the number of crossings, more effective will be the drawing of the

graph. But in many cases crossings cannot be avoided. For example in bipartite graph there is a restriction to set all the vertices in two lines. This problem is an approach to find more efficient layout with less crossings by removing this restriction.

Given a bipartite graph $G = (A \cup B, E)$. In one layer one cluster model vertex set A is placed freely on a line L and vertex set B is set freely in a region R. Objective of this problem is to find a one layer one cluster drawing with a minimum number of edge crossings. This problem is NP- complete [ZSE] and has considerable applications in social networks, spam reduction on the web, in meta-search etc.

4.5 Crossing Minimization Problem of Drawing Bipartite Graph in Two Clusters -

This problem is a variant of the problem defined in Section 3.4. In this problem, for a given bipartite graph $G = (A \cup B, E)$, vertex sets A and B are placed in two disjoint regions R_1 and R_2. In each region, vertices can be set freely. This problem consists of finding a two cluster drawing for a given bipartite graph with minimum number of crossings. This problem is also NP-complete [ZSE].

4.6 The Shortest Path Problem on Networks with Intuitionistic Fuzzy Edge Weights -

The shortest path problem is the problem of finding the shortest path in a graph from one vertex to another. “Shortest” may be least number of edges, least total edge weight etc.

In traditional shortest path problem the arc lengths (edge weights) of a network take precise (crisp) values. But in real life situations the arc length may represent cost, time etc. There are many cases where a unique number cannot be assigned to edge weights, because of cost, time variation. Hence an intuitionistic fuzzy weight may be assigned to the edges [CBG]. The objective of this problem is to find a shortest path in a given network (or graph) in which a intutionistic fuzzy weight has been assigned to each edge of the network.

5. Literature Review-

Exact methods and approximation algorithms are available in literature for some of the problems mentioned above. Researchers have also focused on designing metaheuristics for some problems whereas for some problems there is no exact or metaheuristic algorithm available in literature. Since the focus of this work would be on designing and implementing metaheuristics for solving graph theoretic optimization problems in crisp and fuzzy environment, therefore heuristic algorithms which have been developed for above mentioned problems are surveyed here. Besides, available exact results for some problems mentioned above will be used in order to test the performance of our algorithms, therefore in the following sections, an outline of the relevant work done on these problems so far is given briefly.

The bandwidth coloring problem is NP-hard since it generalizes the classical vertex colouring problem [GJ1]. Marti et al. [MGD] described both memory-based and memory-less methods to solve the bandwidth coloring problem. In particular he proposed new constructive and improvement methods based on tabu search and GRASP. Lai and Lu [LL] in 2013 proposed a multistart iterated tabu search (MITS) algorithm for solving bandwidth coloring problem and bandwidth multi-coloring problem. MITS algorithm exhibits several distinguishing features, such as integrating an iterated tabu search (ITS) algorithm with a multistart method and a problem specific perturbation operator. In 2014 Lai et al. [LLHGX] also presented a path relinking algorithm which combines a population based relinking method and a tabu search based local search procedure. Matic et al. [MKF] developed a variable neighborhood search algorithm for solving bandwidth coloring problem. There is still a wide scope for designing and developing other metaheuristics for improvement in the results and for reducing time complexity.

Sugiyama [STT] proposed the so-called barycentric method for arc crossing minimization in hierarchical digraphs. This is probably the most established method for crossing reduction. Since crossing minimization is an NP-complete problem [GJ2], many exact and metaheuristic algorithms have been proposed to minimize number of crossings. In spite of practical significance, very little attention has been paid to the incremental variant of the graph drawing problem. Marti and Estruch (2001) [ME] proposed a greedy randomized adaptive search procedure for incremental bipartite graphs. A variable neighborhood decent for incremental graph drawing of hierarchical graphs has been proposed in 2017 [OGLDM].

Median graph problem is NP-complete [HC]. Earlier approximate approaches to find the generalized median graph include greedy [HW] and genetic [JMB] algorithms. Ferrer (2008) [F] and Ferrer et al. (2009) [FVS1] developed exact approaches for dealing with graphs with maximum 25 nodes. Ferrer et al. [FVS2] also proposed an approximate algorithm based on the genetic search to compute the median graph.

Crossing minimization problems of drawing bipartite graph in one layer one cluster and two clusters are introduced and proved to be NP-complete in 2005 by Lanbo et al. [ZSE]. But till date no metaheuristic has been proposed for these problem. So there is a wide scope for developing and designing metaheuristics for the problems.

For the shortest path problem on networks with intuitionistic fuzzy edge weights, no metaheuristic has been developed so far. Although some exact methods based on ranking technique [SH, B2] are introduced.

6. Objectives –

Since, for most of the problems described in Section 4, no metaheuristic has been developed except for bandwidth coloring problem, incremental graph drawing problem and generalized median graph problem, so there is a wide scope for designing and development of metaheuristics for these problems. Moreover, for bandwidth coloring problem and incremental graph drawing problem only single solution based metaheuristics exist. So one can explore and identify

population based metaheuristics for these problems. Further, for generalized median graph problem only genetic algorithm is proposed, so there is a room for designing of other metaheuristics (both single solution based and population based). To evaluate the performance of the heuristics that we propose in our work, statistical analysis on benchmark problems/random graphs may also be done. Finally, the objectives we intend to consider in our research work are summarized as follows:

1. Exploration and identification of applications of metaheuristics to the graph theoretic optimization problems mentioned in Section 4.
2. Developing problem specific (knowledge based) construction heuristics in order to generate good initial solutions specially needed for the population based metaheuristics.
3. Problem dependent genetic operators such as crossover, mutation etc may also be designed as existing operators usually are not efficient enough to solve specific problems.
4. Designing of local improvement strategies and neighborhood structures will be another focus area as they play a key role in the performance of a metaheuristic.
5. Statistical evaluation of developed construction heuristics/operators/techniques on benchmark problems/random graphs/standard graphs with known results (if available).
6. Exploration of some other optimization problems concerned with graphs having real life applications.

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